Signals and Systems

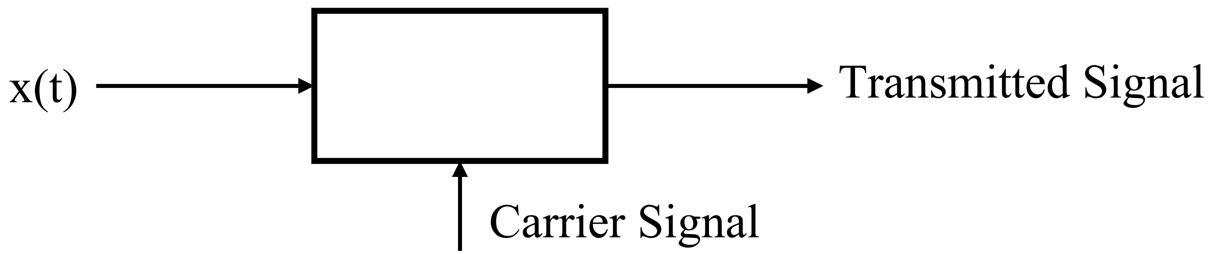
Fall 2003

Lecture #15

28 October 2003

1. Complex Exponential Amplitude Modulation
2. Sinusoidal AM
3. Demodulation of Sinusoidal AM
4. Single-Sideband (SSB) AM
5. Frequency-Division Multiplexing
6. Superheterodyne Receivers

The Concept of Modulation

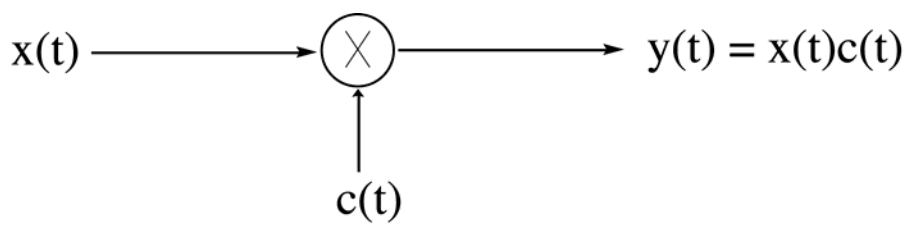


Why?

- More efficient to transmit E&M signals at higher frequencies
- Transmitting multiple signals through the same medium using different carriers
- Transmitting through "channels" with limited passbands
- Others...

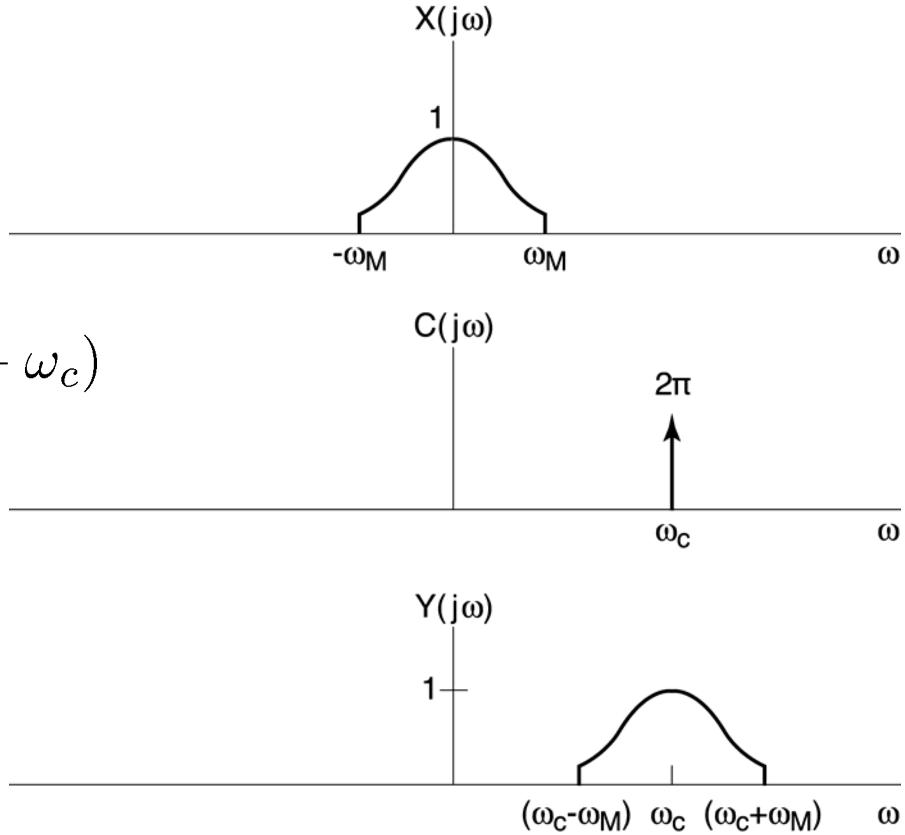
How?

- *Many* methods
- Focus here for the most part on *Amplitude Modulation* (AM)

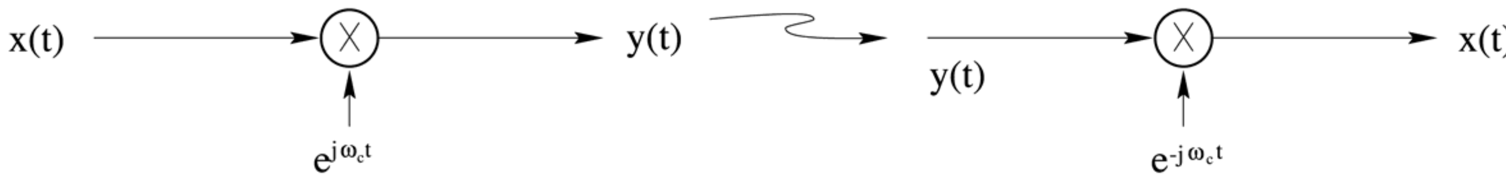


*A*mplitude *M*odulation (AM) of a Complex Exponential Carrier

$$
\begin{aligned}
c(t) &= e^{j\omega_c t}, \quad \omega_c - \text{carrier frequency} \\
y(t) &= x(t)e^{j\omega_c t} \\
Y(j\omega) &= \frac{1}{2\pi} X(j\omega) * C(j\omega) \\
&= \frac{1}{2\pi} X(j\omega) * 2\pi\delta(\omega - \omega_c) \\
&= X(j(\omega - \omega_c))
\end{aligned}
$$



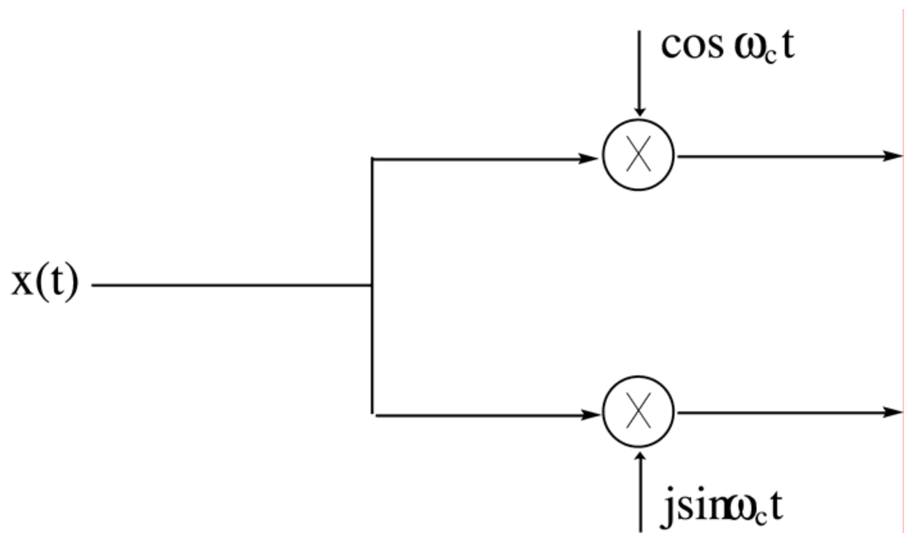
Demodulation of Complex Exponential AM



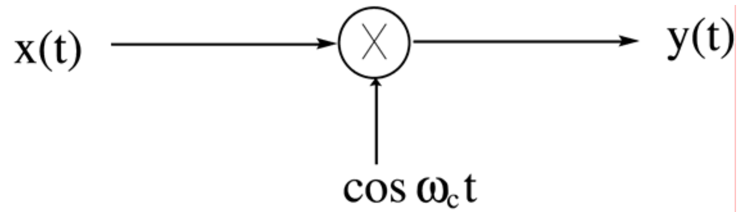
$$e^{j\omega_c t} = \cos\omega_c t + j\sin\omega_c t$$

$$\Downarrow$$

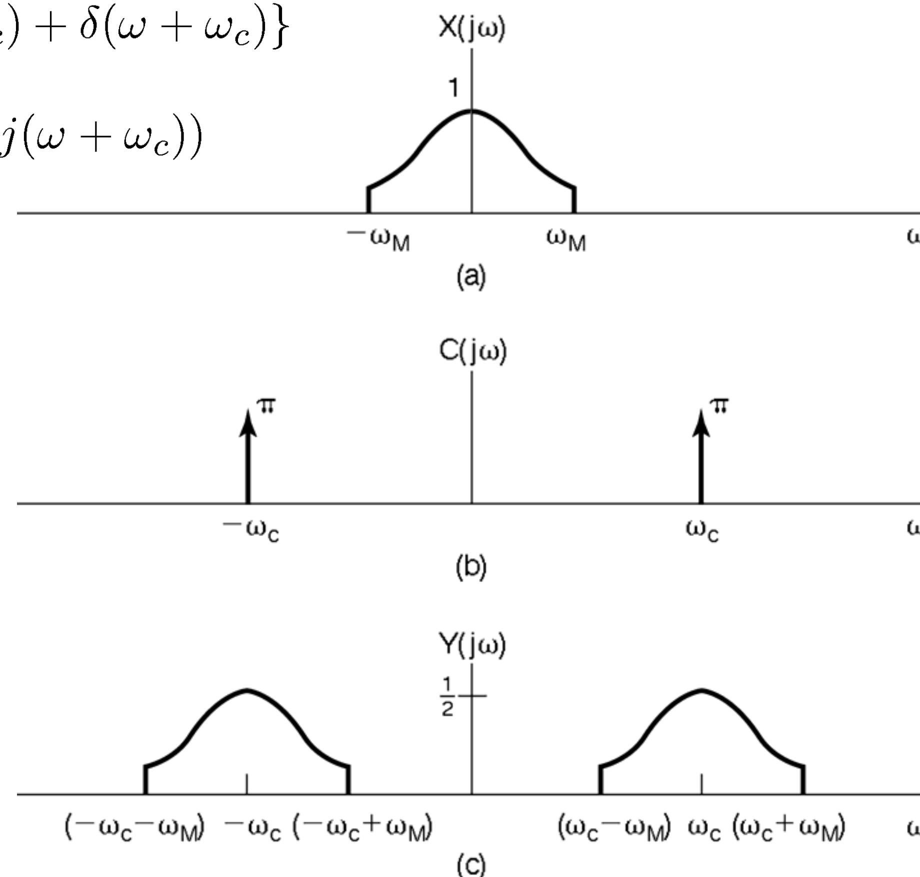
Corresponds to two separate modulation channels (quadratures) with carriers 90° out of phase



Sinusoidal AM



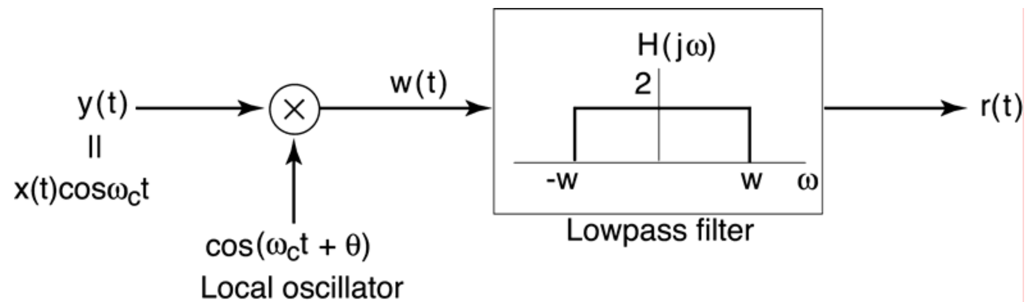
$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * \pi\{\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\}$$
$$= \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$



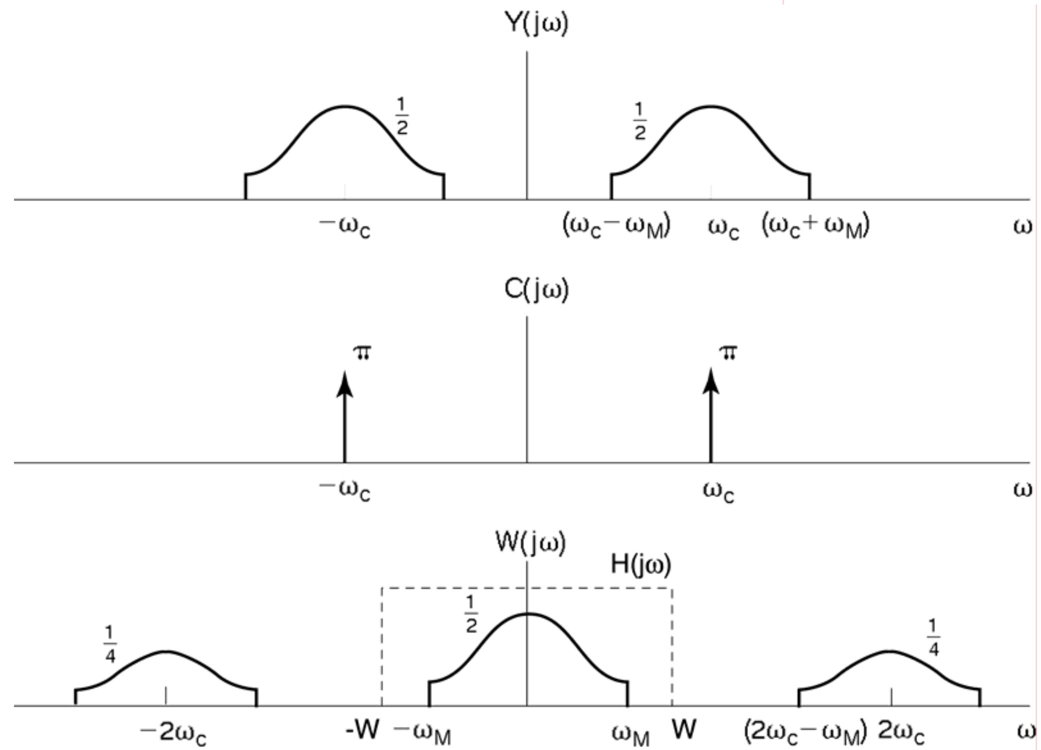
Drawn assuming

$\omega_c > \omega_M$

Synchronous Demodulation of Sinusoidal AM



Suppose θ = 0 for now,
⇒ Local oscillator is in phase with the carrier.



Synchronous Demodulation in the Time Domain

$$w(t) = y(t)\cos\omega_c t = x(t)\cos^2\omega_c t = \frac{1}{2}x(t) + \underbrace{\frac{1}{2}x(t)\cos 2\omega_c t}_{\text{High-frequency signals filtered out by the LPF}}$$

Then $$r(t) = x(t)$$

Now suppose there is a phase difference, *i.e.* θ ≠ 0, then

$$\begin{aligned} w(t) &= y(t)\cos(\omega_c t + \theta) = x(t)\cos\omega_c t\cos(\omega_c t + \theta) \\ &= \frac{1}{2}x(t)\cos\theta + \underbrace{\frac{1}{2}x(t)(\cos(2\omega_c t + \theta))}_{\text{HF signal}} \end{aligned}$$

Now $$r(t) = x(t)\cos\theta$$

Two special cases:

1) $\theta = \pi/2$, the local oscillator is 90° out of phase with the carrier, $\Rightarrow r(t) = 0$, signal unrecoverable.
2) $\theta = \theta(t)$ — slowly varying with time, $\Rightarrow r(t) \cong \cos[\theta(t)] \cdot x(t)$, $\Rightarrow$ time-varying "gain".

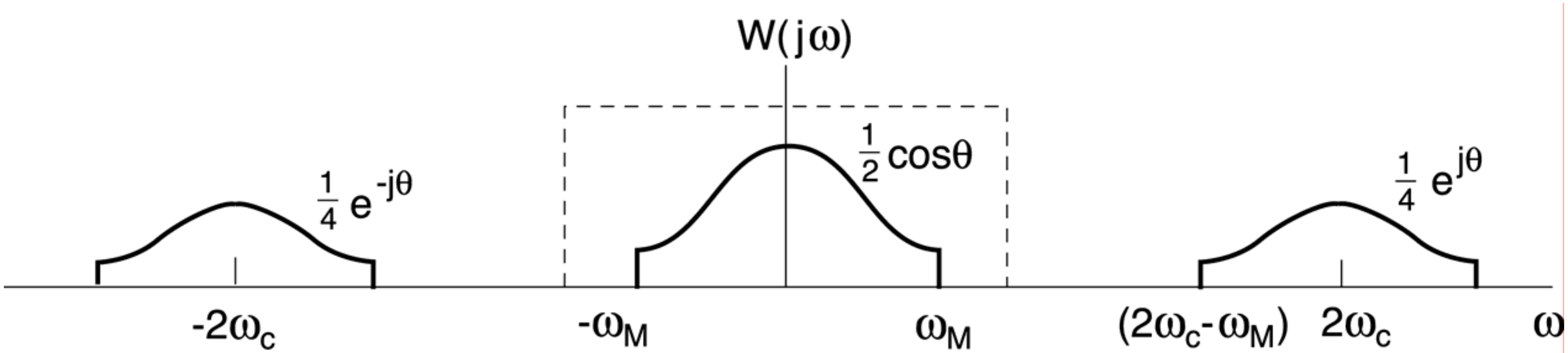
Synchronous Demodulation (with phase error) in the Frequency Domain

Demodulating signal –
has phase difference θ w.r.t.
the modulating signal

$$\cos(\omega_c t + \theta) = \frac{1}{2}e^{j\theta}e^{j\omega_c t} + \frac{1}{2}e^{-j\theta}e^{-j\omega_c t}$$

$$\Downarrow \mathcal{F}$$

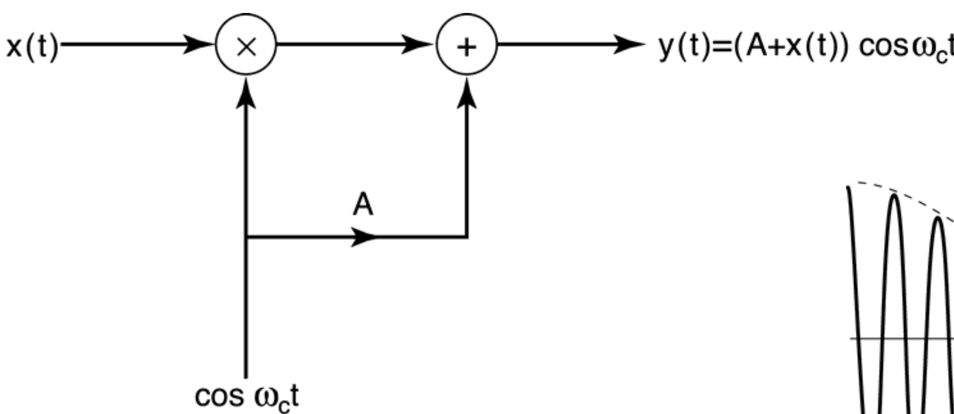
$$\pi e^{j\theta}\delta(\omega - \omega_c) + \pi e^{-j\theta}\delta(\omega + \omega_c)$$

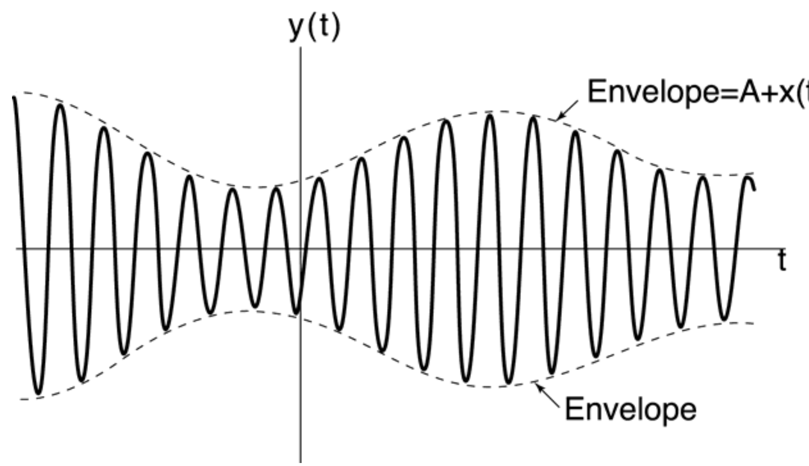


Again, the low-frequency signal $(\omega < \omega_M) = 0$ when $\theta = \pi/2$.

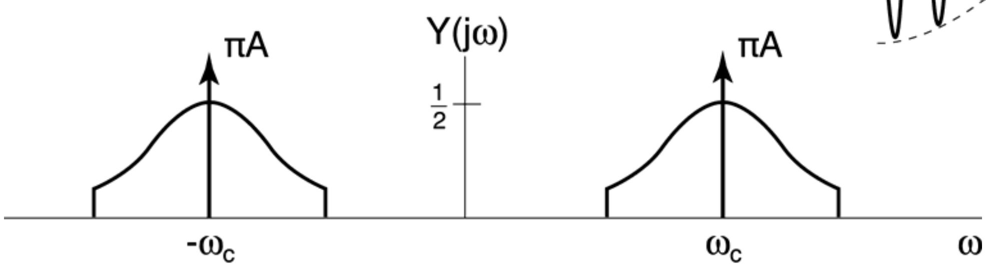
Alternative: Asynchronous Demodulation

- Assume $\omega_c >> \omega_M$, so signal envelope looks like $x(t)$
- Add same carrier with amplitude A to signal





Time Domain

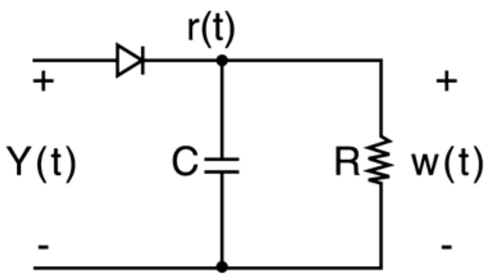


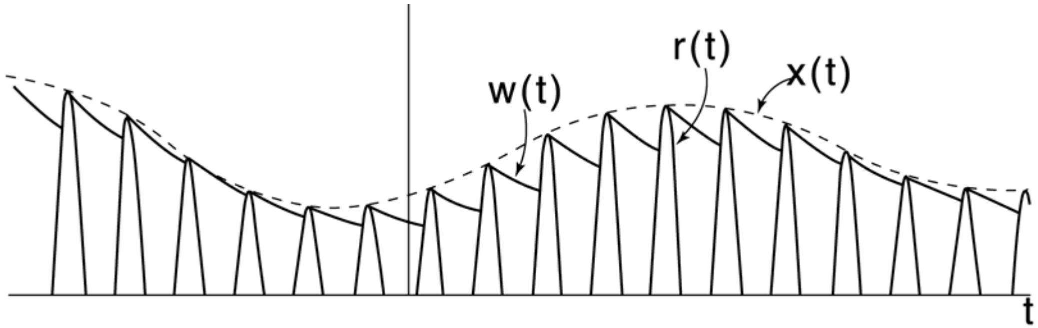
Frequency Domain

$A = 0 \Rightarrow$ DSB/SC (Double Side Band, Suppressed Carrier)

$A > 0 \Rightarrow$ DSB/WC (Double Side Band, With Carrier)

Asynchronous Demodulation (continued)
Envelope Detector





In order for it to function properly, the envelope function must be positive for all time, *i.e.* $A + x(t) > 0$ for all t.

Demo: Envelope detection for asynchronous demodulation.

Advantages of asynchronous demodulation:

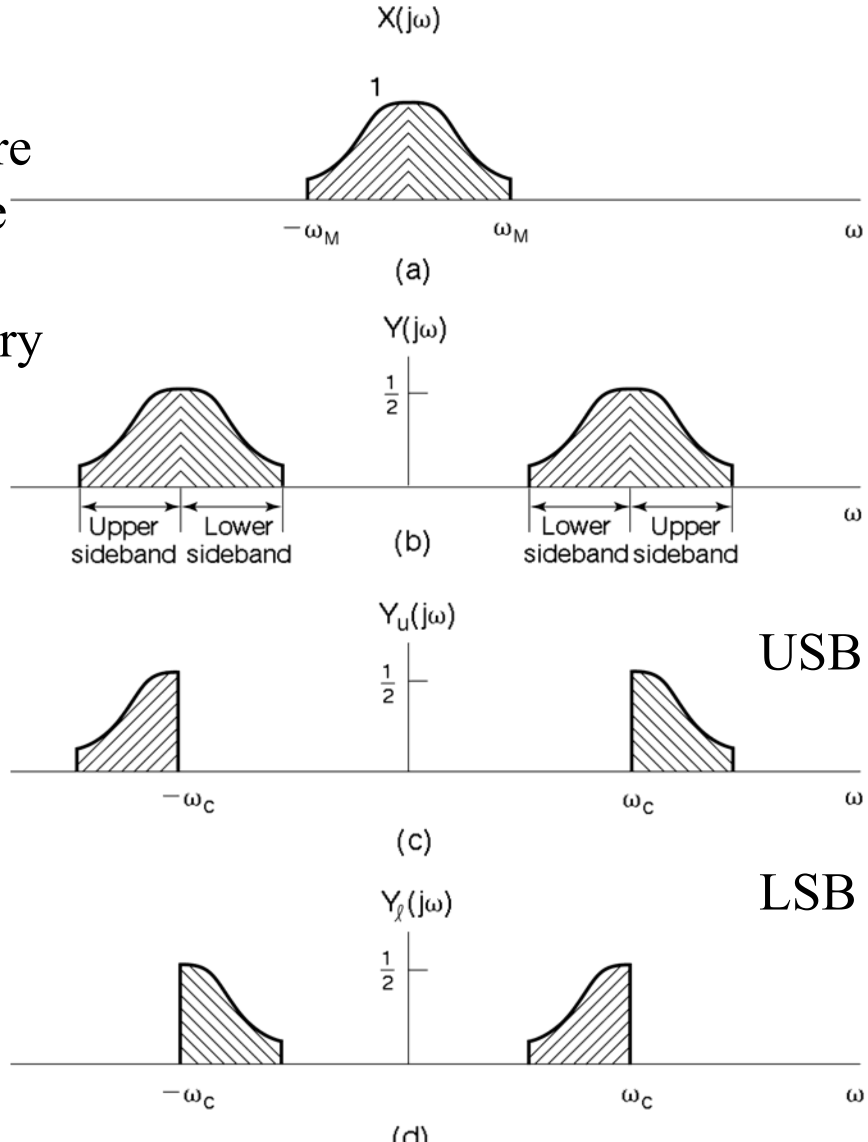
— Simpler in design and implementation.

Disadvantages of asynchronous demodulation:

— Requires extra transmitting power $[A\cos\omega_c t]^2$ to make sure $A + x(t) > 0 \Rightarrow$ Maximum power efficiency = 1/3 (P8.27)

Double-Sideband (DSB) and Single-Sideband (SSB) AM

Since $x(t)$ and $y(t)$ are *real*, from conjugate symmetry both *LSB* and *USB* signals carry exactly the same information.



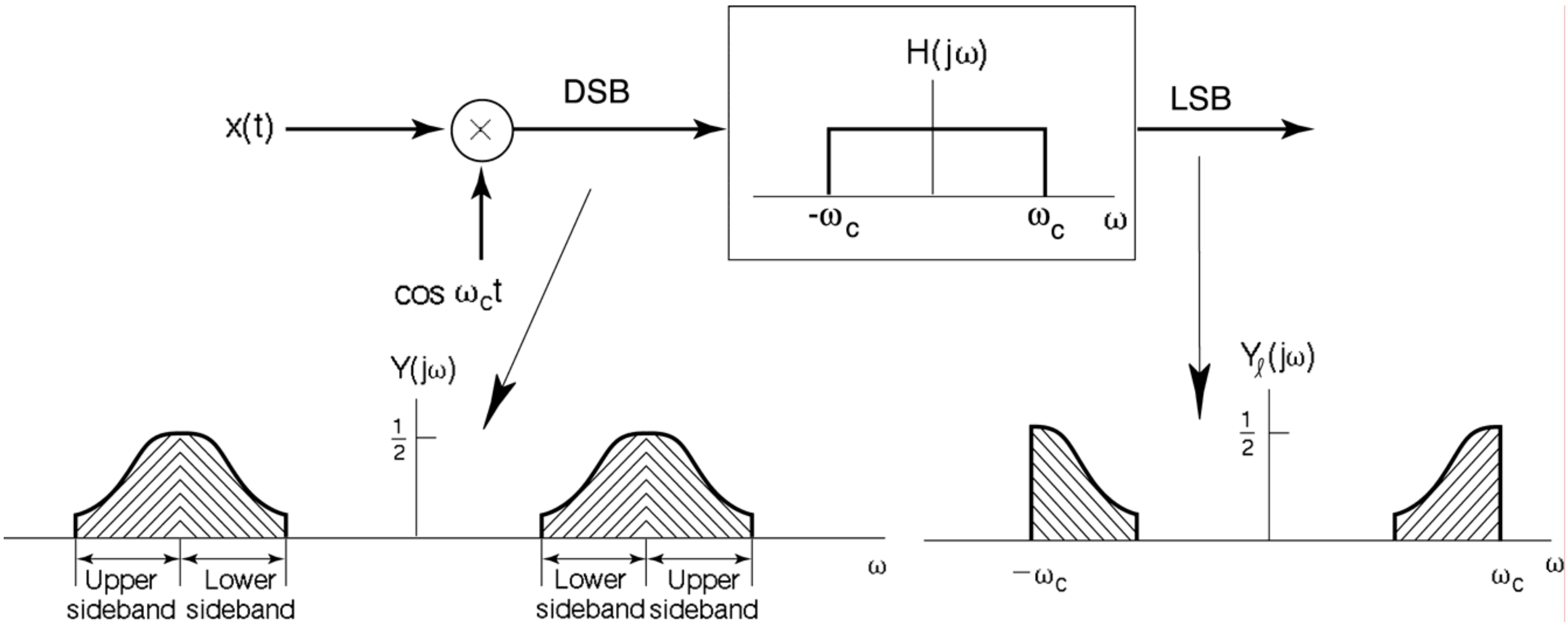
DSB, occupies $2\omega_M$ bandwidth in $\omega > 0$.

USB

LSB

Each sideband approach only occupies ω_M bandwidth in $\omega > 0$.

Single Sideband Modulation

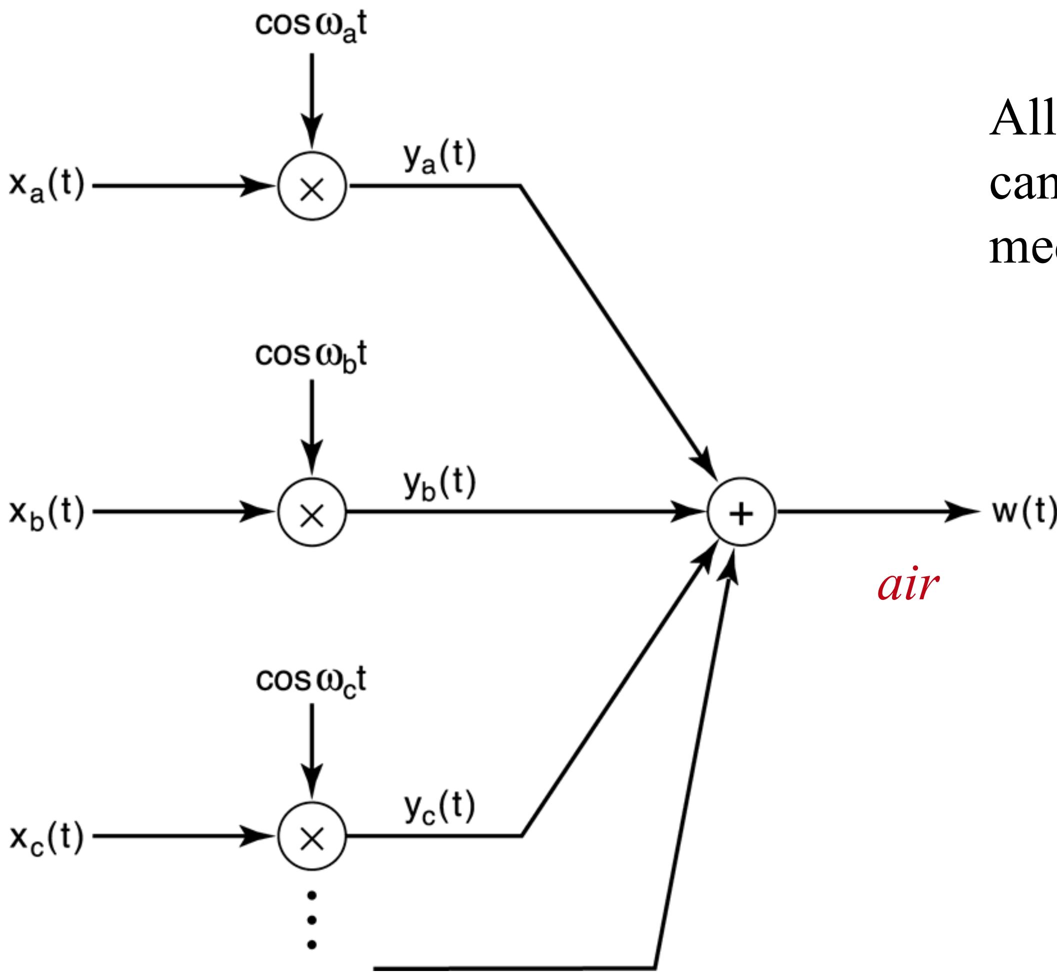


Can also get SSB/SC

or SSB/WC

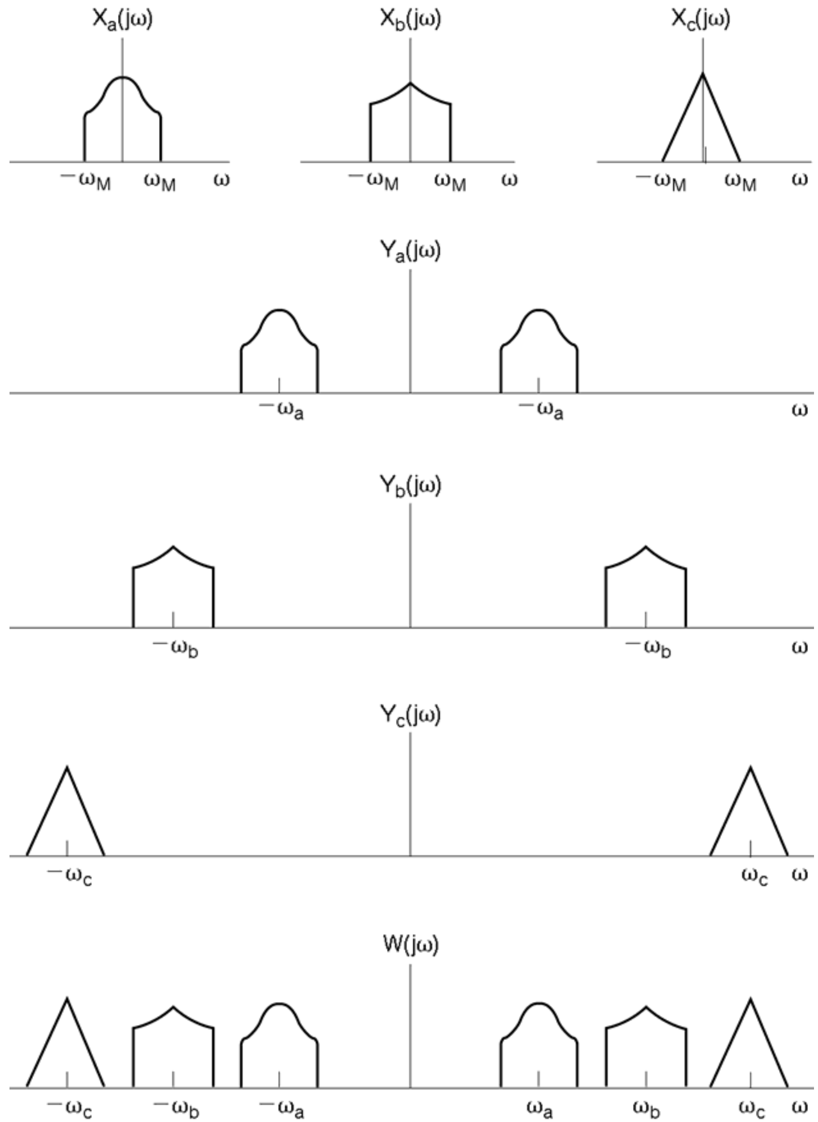
Frequency-Division Multiplexing (FDM)

(Examples: Radio-station signals and analog cell phones)



All the channels can share the same medium.

FDM in the Frequency-Domain



"Baseband" signals

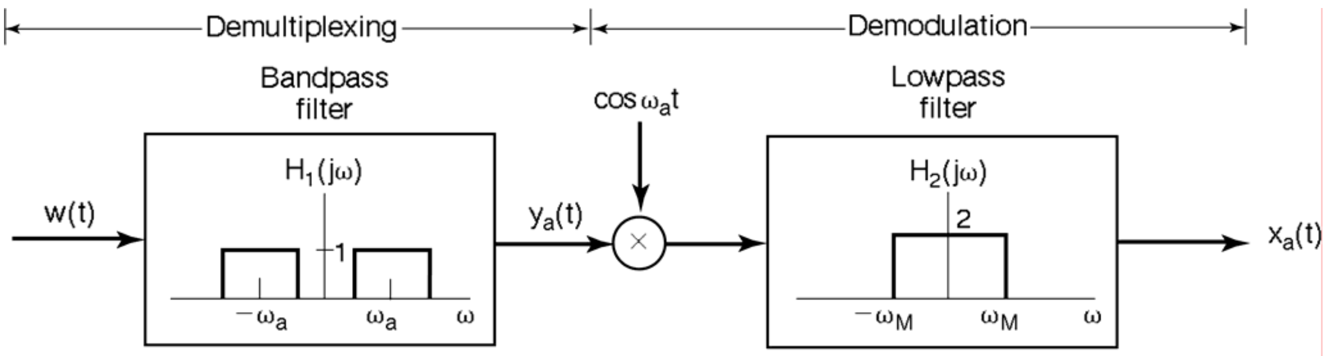
Channel a

Channel b

Channel c

Multiplexed signals

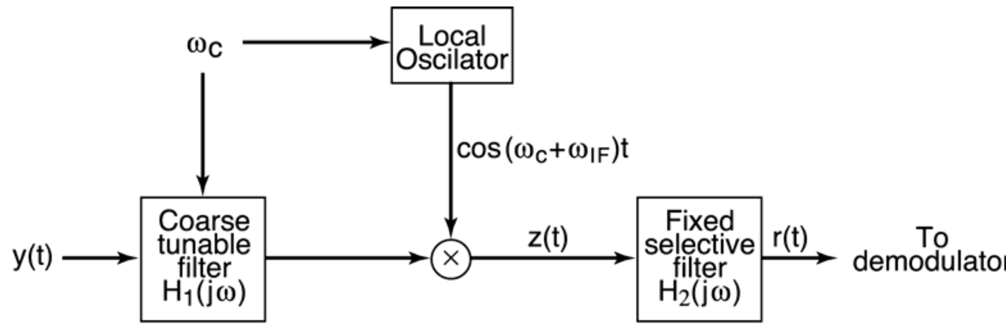
Demultiplexing and Demodulation

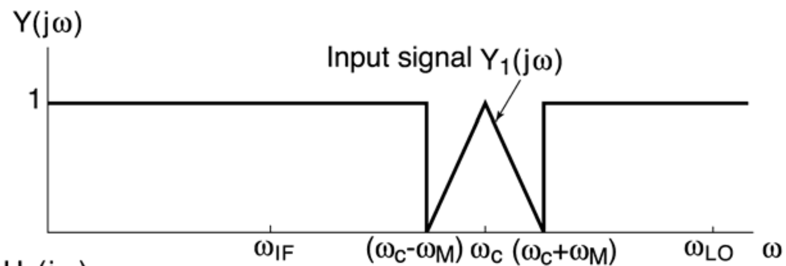


ω_a needs to be tunable

- Channels must not overlap ⇒ Bandwidth Allocation
- It is difficult (and expensive) to design a highly selective bandpass filter with a tunable center frequency
- Solution – Superheterodyne Receivers

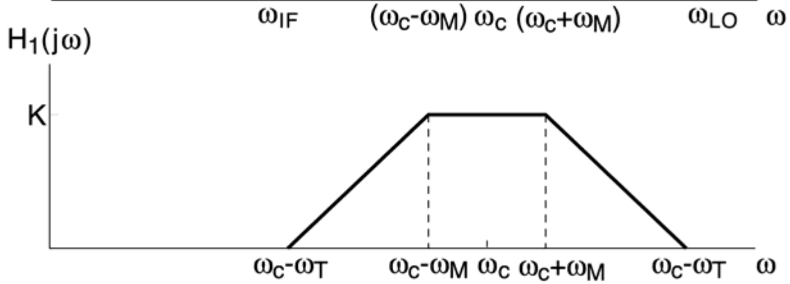
The Superheterodyne Receiver



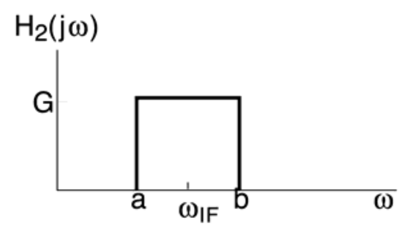


$$\text{AM,}\quad \frac{\omega_c}{2\pi} = 535 - 1605\,kHz \;\text{— RF}$$

$$\text{FCC:}\quad \frac{\omega_{IF}}{2\pi} = 455\,kHz \;\text{— IF}$$



Coarse tunable filter



Fixed BPF

Operation principle:

— Down convert from ω_c to ω_{IF}, and use a coarse tunable BPF for the front end.

— Use a sharp-cutoff *fixed* BPF at ω_{IF} to get rid of other signals.